ZESZYTY NAUKOWE UNIWERSYTETU SZCZECIŃSKIEGO nr 854
Finanse, Rynki Finansowe, Ubezpieczenia nr 73 (2015)

s. 325–339

Rate of Return, Risk – Stimulators of Investment Decisions on a Capital Market

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Summary: The article presents the problems of quantitative measures which may assist the investor, especially a risk-averse one, in making decisions regarding a capital market. The article also contains an example of the process of making investment decisions. Mathematical considerations are also conducted which can be summarized as determining a set of effective portfolios, with a specific numerical example. A minimum risk portfolio may be selected by the investor from such a set.

Keywords: randomness, normal distribution, profit, risk

Introduction

An investor allocating his/her capital savings into a given type of securities makes his/her choice on the basis of expected income; therefore he/she is acting based on the foreseen increase of the price of a given type of securities (Bradley 1992; Duraj 1997). In Markowitz's classic theory it is assumed that the future profit rate may be determined (approximately) through return on investment (profit) obtained in the future. Therefore, when buying in order to build a proper investment portfolio we are dealing with profit expected in the future. It is based not only on the choice made, which may be more or less accurate, but also on the condition of the market which will take place in the future. Rate of return is, in essence, profit, which is realized in due time, hence it is a future rate of return, a function of a random variable of profit or loss (which in turn is tied with the uncertainty of its realization).[1] Such a statement implies the necessity of knowing its probability dispersion. (Tarczyński 1997: 13).

Random rate of return and its probability distribution in investment decisions

Rate of return as such is a univariate, random discrete variable. This variable is often approximated as a continuous function, which is especially useful in portfolio risk analysis.

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[1] In a long-term prognosis of rate of return an investment portfolio may not show randomness but the characteristics of a trend (Tymiński 2013: 27).

Probability distribution for a random variable R (rate of return) can be assigned as:

1. A set of parameter pairs or a cumulative distribution function (Hellwig 1998; Jajuga 2000):
   – discrete:

$$(R_i, p_i),\ (i = 1, ..., n), \tag{1}$$

$$F(R) = \sum_{R_i < R} p_i, \tag{2}$$

   where: R_i – i-th possible rate of return; p_i – probability of occurrence of such a natural state, that will allow achieving the i-th rate of return,
   – continuous density function $f(R)$ for continuous distribution – accordingly a: cumulative distribution function, a value for the expected rate of return and a standard deviation:

$$F(R) = \int_{-\infty}^{R} f(R)dR \tag{3}$$

$$E(R) = \int_{-\infty}^{\infty} Rf(R)dR \tag{4}$$

   and:

$$\sigma = \sqrt{\int_{-\infty}^{\infty} [R - E(R)]^2 f(R)dR} \tag{5}$$

2. Moments of a discrete random variable:
   – value of an expected variable R, which is referred to in literature as a typical 1st moment for a discrete probability distribution:

$$E(R) = \sum_{i=1}^{n} p_i R_i\ , \tag{6}$$

   where: n – number of possible values for rate of return (risk variable);
   – standard deviation, which is the square root of a variance (also referred to as the central moment of the 2nd rank):

$$\sigma = \sqrt{\sum_{i=1}^{n} p_i [R_i - E(R)]^2} \tag{7}$$

3. Mean deviation:
   – continuous density function:

$$d(R) = \int_{-\infty}^{\infty} | R - E(R) | f(R)dR \tag{8}$$

– discrete random variable:

$$d(R) = \sum_{i=1}^{n} p_i \,|\, R_i - E(R) \,|. \tag{9}$$

It needs to be underlined, that the connection between a standard deviation and a mean deviation, as well as a mean semi-deviation, standard semi-deviation (which is an expression of the negative concept of risk) may be explained mathematically in the following way.

In decisions made by investors on capital markets we have to do with strategies based on the criteria of predicted risk and profit, which can be diluted down to maximization of profit and minimization of risk through the appropriate construction of an investment portfolio. In such a setting it is assumed that the risk prognosis is the value expected E(R), which is a point prediction. It remains obvious, that the higher the value of the prediction, the more favourable the decision of the investor pertaining to the optimization of a constructed portfolio. *R,* however remains a random variable, because the outcome as such may vary from the prediction (Pawłowski 1982; Nowak (ed.) 1998). Hence, the risk that a given prediction is not fulfilled needs to be taken into account. In the portfolio theory, within the process of predicting the expected rate of return, risk has two main meanings (Czerwiński 1982; Jajuga 2009):

- possibility of occurrence of a result misaligned with expectations – it makes no difference whether the result is favourable or unfavourable, but it becomes obvious that the prediction itself was inaccurate,
- the possible loss that may be incurred or other unpleasant, unexpected occurrence (for example, the necessity of short sale abandonment, withdrawing a company from the market, due to its stock being purchased by an investor) – here apart from the most popular risk measures, which are the standard deviation and variance, we also apply the negative standard semi-deviation and the negative semi-variance (they are the measures of loss incurred).

The next group of measures consists of the mean deviation and mean semi-deviation.

It has to be noted though, that in the case of normal distribution these measures are equivalent to the standard deviation and the standard semi-deviation, because they are proportional to them. Thus, if *R* has a normal distribution characterized by the parameters *m* i *σ*, that is $R \sim N(m, \sigma)$, then (Jaworski, Micał 2005: 277):

$$\sigma^2(R) = \sigma^2,\ \ \sigma(R) = \sqrt{\sigma^2},\ \ s\sigma^2(R) = 1/2\sigma^2,\ \ s\sigma(R) = \frac{\sqrt{2}}{2}\sigma$$

and the mean deviation

$$d(R) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} |\, x - m \,| \exp\left[-\frac{(x-m)^2}{2\sigma^2} \right] dx = \frac{2\sigma_R}{\sqrt{2\pi}}; \tag{10}$$

The formula for $d(R)$ is derived by way of computing an improper integral: (Łubnicki et al. 1996)

$$\frac{1}{\sigma_R\sqrt{2\pi}}2\int(R-m)\exp\left[-\frac{(R-m)^2}{2\sigma_R^2}\right]dR=\left[\begin{array}{l}\frac{(R-m)^2}{2\sigma_R^2}=t\\ \frac{-2(R-m)}{2\sigma_R^2}dR=dt\\ (R-m)dR=-\sigma_R^2dt\\ \text{Dla } R\in\langle m,\infty)\text{ mamy } t\in\langle 0,\infty)\end{array}\right]=$$

$$-\frac{2}{\sigma_R\sqrt{2\pi}}(-\sigma_R^2)\int_{-\infty}^{0}e^t dt=\frac{2\sigma_R}{\sqrt{2\pi}}\lim_{\alpha\to-\infty}e^t\,|_\alpha^0=\frac{2\sigma_R}{\sqrt{2\pi}}\lim_{\alpha\to-\infty}(e^0-e^\alpha)=$$

$$=\frac{2\sigma_R}{\sqrt{2\pi}}\cdot(1-0)=\frac{2\sigma_R}{\sqrt{2\pi}},$$

and

$$sd^-(R)=E(R-m)^-)=\frac{-1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{0}R\exp\left(-\frac{R^2}{2\sigma^2}\right)dR=\frac{\sigma_R}{\sqrt{2\pi}} \tag{11}$$

$$s\sigma^-=\sqrt{\sum_{i=1}^{n}p_i[R_i-E(R)^-]^2}$$

for

$$R_i-E(R)^-=\begin{cases}R_i-E(R) & \text{gdy } R_i-E(R)<0\\ 0 & \text{gdy } R_i-E(R)\geq 0\end{cases} \tag{12}$$

where:

$\sigma^2(R)$	– rate of return variance (σ^2),
$\sigma(R)=\sqrt{\sigma^2}$	– standard deviation,
$s\sigma^2(R)$	– negative semi-variance,
$s\sigma^{2-}(R)=\sqrt{s\sigma^2}$	– negative semi-deviation,
$d(R)$	– mean deviation,
$sd^-(R)$	– negative mean semi-deviation,
p	– level of probability.

Formula (11) for $sd^{-}(R)$ is derived in a similar way as formula (10):

$$-\frac{1}{\sigma_R\sqrt{2\pi}}\int_{-\infty}^{0} R\exp\left(-\frac{R^2}{2\sigma_R^2}\right)dR=\left[\begin{array}{l}\frac{-R^2}{2\sigma_R^2}=t\\ \frac{-2R^2}{2\sigma_R^2}=dt\\ RdR=-\sigma_R^2 dt\\ \text{Dla } R\in(-\infty,0\rangle \text{ mamy } t\in(-\infty,0\rangle\end{array}\right]=$$

$$-\frac{1}{\sigma_R\sqrt{2\pi}}\int_{-\infty}^{0}(-\sigma_R^2)e^t dt=\frac{\sigma_R}{\sqrt{2\pi}}\lim_{\alpha\to-\infty}\int_{\alpha}^{0}e^t dt=\frac{\sigma_R}{\sqrt{2\pi}}\lim_{\alpha\to-\infty}e^t\,|_{\alpha}^{0}= \tag{13}$$

$$=\frac{\sigma_R}{\sqrt{2\pi}}\lim_{\alpha\to-\infty}(e^0-e^{\alpha})=\frac{\sigma_R}{\sqrt{2\pi}}(1-e^{-\infty})=\frac{\sigma_R}{\sqrt{2\pi}}\cdot(1-0)=\frac{\sigma_R}{\sqrt{2\pi}}$$

A comparison of formulas (10) and (11) shows that: $d(R) = 2sd^{-}(R)$.

In the case of normal distribution in risk analyses one may thus apply interchangeably the mean deviation in place of standard deviation also in other areas of the operation of a given company (Krawczyk 2001). Amongst the distributions of random variables in statistics and other related sciences, for example in the problematic aspects of a portfolio, in the events of uncertainty which occur in long term predictions pertaining to investor decisions, the normal distribution, known as the Gauss distribution, is most often used (Tinter, Sengupta 1972; Sadowski 1969).

Example

A sample analysis of the formation of the rate of return was carried out in the situation of an uncertain stock portfolio. The analysis of a stock portfolio of a selected stock company within a period of five years (The Parkiet Newspaper, 1999–2009), carried out on investor's commission, produced an amount of information pertaining to monthly rate of return (Table 1).

A graphic illustration of the distribution of monthly rates of return has been provided in Figure 1. As can be seen in Figure 1 the greatest probability can be observed at rates of return 3.0 and 3.4, while for rates of return whose value is less than 3.0 and greater than 3.4 one can observe an even increase or decrease of the value of probability. Such a distribution can be considered as similar to normal distribution (despite being slightly dextral).

Table 1

Data on monthly rates of return

N.	Interval of rate of return value (%)	Number of favourable cases n_i	Probability rates of return: $p_i = \frac{n_i}{\sum n_i}$ realization	$\sum_i p_i$ cumulative	Value of the cumulative distribution function $F(R)$
1.	1.20–1.60	1	1/60	1/60	0
2.	1.60–2.00	2	2/60	3/60	58/60
3.	2.00–2.40	4	4/60	7/60	56/60
4.	2.40–2.80	8	8/60	15/60	52/60
5.	2.80–3.20	14	14/60	29/60	46/60
6.	3.20–3.60	12	12/60	41/60	48/60
7.	3.60–4.00	7	7/60	48/60	53/60
8.	4.00–4.40	6	6/60	54/60	54/60
9.	4.40–4.80	3	3/60	57/60	57/60
10.	4.80–5.20	2	2/60	59/60	58/60
11.	5.20–5.60	1	1/60	1	59/60
12.	>5.60	0	0	1	1

Source: author's own calculations.

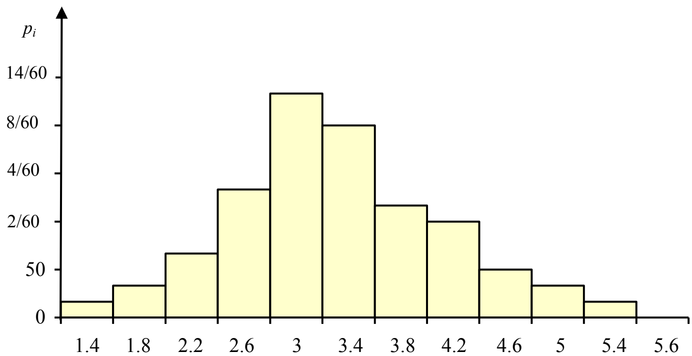

Figure 1. Histogram of rate of return distribution

Source: author's own elaboration.

Should an investor be interested in purchasing stocks of this company with the value of the rate of return no less than 4.0% the probability of fulfilment of this prediction equals (Tab. 1):

$$P(R \geq 4.00) = 1 - F(4.00) = 1 - \frac{53}{60} = 0.12 \cdot$$

It is however too low a probability for the rate of return realization, so the purchase of the stocks of this particular company is too risky. However, if the investor is satisfied with the probability of the realization of the stock purchase $p \geq 0.30$, then we look for the value which meets this requirement. It is simultaneously assumed that presumably it may be a value from the interval <3.2%, 4.0%>, then: $P(3.2 \leq p_R \leq 4.00) = \frac{12}{60} + \frac{7}{60} = \frac{19}{60} = 0.32$ (Tab. 1).

Standard deviation of rate of return is calculated on the basis of formula (7):

$$\sqrt{\sigma_R^2} = \sqrt{\sum_{i=1}^{n} (R_i - E(R))^2 p_i} \tag{14}$$

Assuming that we are most interested in rate of return R from the interval <3.2; 4.0>, then $E(R) = 3.6$, in such a case formula (14) take the following form:

$$\sigma p_R = \sqrt{\sigma_R^2} = \sqrt{\frac{1}{\sum_{i=1}^{n} \overline{p}_i} \sum_{i=1}^{n} (R_i - E(R))^2 p_i} \tag{14a}$$

We obtain (in %):

$$\sigma_R = \sqrt{\frac{1}{\frac{19}{60}} \left(3.2 - 3.6)^2 \cdot \frac{12}{60} + (4.0 - 3.6)^2 \cdot \frac{7}{60} \right)} = \sqrt{\frac{60}{19} (0.4)^2 \cdot \frac{19}{60}} = 0.4 \, .$$

So the standard deviation of the value of rate of return equals ±0.4%. Therefore, the standard deviation of the mean value is equal to:

$$\sigma(\overline{p}_R) = \frac{\sigma_R}{\sqrt{np - 1}} = \frac{0.4}{\sqrt{60 - 1}} = \frac{0.4}{7.68} = 0.05 \, .$$

From these considerations it follows that:

1. A stock that is predicted for purchase by an investor with a value of rate of return i.e. 3.6%, which may be achieved with the probability of 32.0%, meets his/her expectations. An error in the value of rate of return from the moment of its realization shall equal 0.40%. Finally, $E(R) = (3.6 \pm 0.4\%)$.
2. The mean value of rate of return determines the distribution of mean values calculated for a sample. Deviation (dispersion) of the sole mean value, calculated for a sample chosen from all elements may assume the value of ±0.05%.
3. If the standard deviation (σ_R) is the distribution of the respective values of rate of return in regard to the median value R (that is 3.6) then the standard deviation $\sigma(\overline{p}_R)$ of the mean value determines the distribution of respective mean values from the

calculated mean value from the sample. An assumption was made in the processes of the formation of rate of return that were described in the paper so far, it was assumed that the mean expected rate of return is fixed. Such an assumption is often not justified. The value expected alone is subject to low random changes, which are caused by unforeseeable factors. The existence of such an additional variability will partially distort the assessment of the effectiveness of stock transactions on a capital market. Tymiński (2013: 48) proposes that we determine a realistic total deviation through two partial deviations, which he refers to as the standard deviation centered around the mean (expected rate of return) and the standard deviation of the mean centered around the general mean in a given examined process. Therefore, $\sigma = \sigma_R + \sigma(\overline{p}_R)$, which is presented in Figure 2.

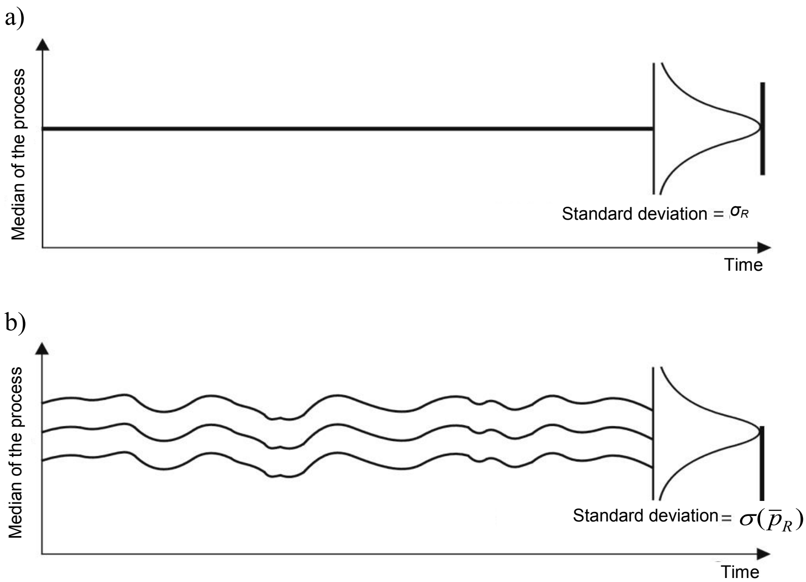

Figure 2. Illustration of the deviation σ

Source: Tymiński (2013): 48.

In the case in which the standard deviation $\sigma(\overline{p}_R)$, is significant, that is when the calculated mean has a high value (in our case it equals "only" 3.6%), such a mean value has to be adjusted within the borders of statistical dispersion. In the presented case it is the value 0.05%. Then one has to include $\sigma(\overline{p}_R)$. A prediction of the rate of return is obtained:

$$R_0 \sim N(E(R); \sigma_R) = N(3.6 \pm 0.4; 0.05).$$

It was assumed in the considerations (due to the low value of $\sigma(\overline{p}_R)$):

$$R \sim N(3.6 \pm 0.4)\,.$$

A normal distribution of rate of return is depicted in Figure 3. The area to the left of the straight line R = 3.6 equals the probability that $R < 3.6\sigma$, while to the right of this line it equals the probability that $R > 3.6\sigma$. Probabilities that $R > 4.0$ and $R < 3.2$ are equal and amount to 0.32 (Figure 3), hence $P(E(R) < 3.2) = P(E(R) > 4.0) = 0.32$

From the characteristics of the cumulative distribution function of normal distribution it follows that:

$$F(-u_0) = 1 - F(u_0), \quad \text{therefore} \quad F\left(\frac{R-3.6)}{0.4}\right) = 1 - F\left(\frac{R-3.6)}{0.4}\right) = 1 - 0.32 = 0.68.$$

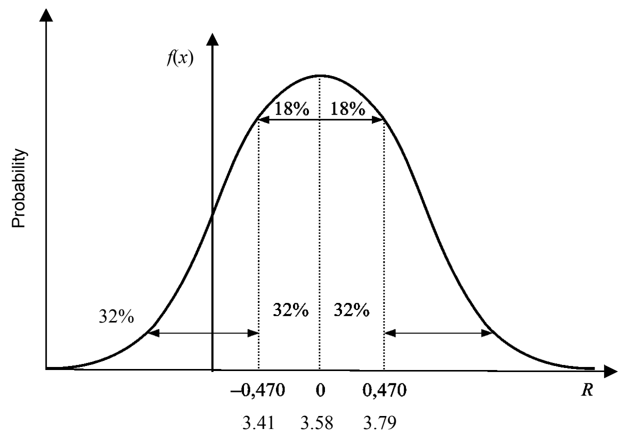

Figure 3. Standardized normal distribution of rate of return

Source: author's own calculations.

Additionally, we may measure the extremities of the interval of variability of rate R, so $\langle R_d, R_g \rangle$ from reliability $u = (R_g - E(R))/\sigma$, that is $R_g = u_0\sigma + E(R)$, and $R_d = u_0\sigma + E(R)$ Then $(R - 3.6)/0.4 = 0{,}47$, so $R_g = 0.47 \cdot 0.4 + 3.6 = 3.79$, and analogically $R_d = -0.47 \cdot 0.4 + 3.6 = 3.41$.

In the case of a significant variability of an observed rate of return an investor should carry out additional analytic research, in order to determine the scope of decision situations. Analytic research should concern the probability of i.e.:

– achieving a rate of return greater than zero,
– occurrence of loss,
– occurrence of loss greater than 0.5%,
– occurrence of loss greater than 3.8%.

It also has to be determined how many standard deviations there are for each of the values considered for profit above the mean value. To calculate these probabilities we apply a formula for standardizing rate of return (Figure 4):

1. $R_{s(1)} = \dfrac{0.0-3.6}{0.4} = -9.0,$ to $P(R>0) = P(R_{s(1)} > -9 = F(9) \approx 1,$ which means that the probability of achieving a rate of return greater than 0.0 is almost equal 1.0. The event is therefore certain (zero rate of return is located at a distance of 9.0 standard deviations from the mean).
2. $R_{s(1)} = \dfrac{0.0-3.6}{0.4} = -0.9,$ or the probability that a loss shall occur equals zero,
3. $R_{s(1)} = \dfrac{3.6-0.0}{0.4} = 0.9,$ which means that the probability of a loss of 0.5% is infinitesimally small, almost equal to zero (as in the case of 1. and 2.).
4. $R_{s(3)} = \dfrac{0.5-3.6}{0.4} = -7.75,$ hence the probability of fulfilment (realization) of this value ($R_{s(4)}$) shall equal 28.1% (because: 1 – 0.719 = 0.281).

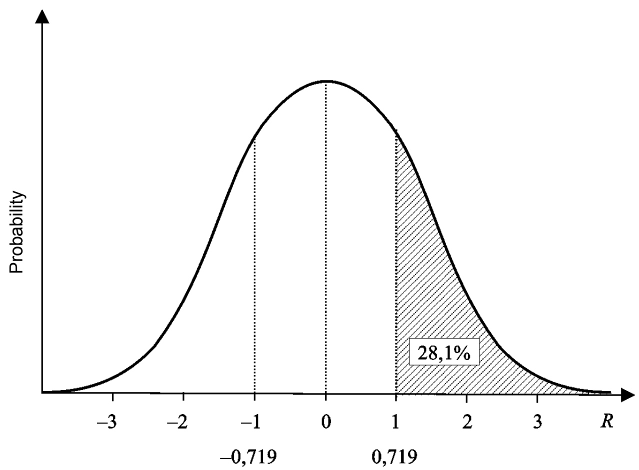

Figure 4. Most probable realization of rate of return

Source: author's own calculations.

The conducted decision analysis of the formation of the value of rate of return for a company stock that the investor wishes to purchase in retrospect (*ex post* prognosis) has shown a risky decision situation which means that a risk-averse investor should refrain from purchasing. However, this situation usually implies a further research process in the area of risk methodology and its measurement, especially in the case of a portfolio analysis conducted for a long term investment strategy (Głowacki 2001; Dworecki 1989 also Tymiński 2013). A minimum risk portfolio should be preferred here.

Minimum Risk Portfolio (MRP)

For an investor guided by an aversion to risk, minimum risk portfolios are important. On the basis of research carried out by Jaworski and Micał (2005: 293 and further), as well as Tymiński's (2013: 75) we know that:

1. A set of probabilities M(P_0) of selecting portfolios by an investor is limited by the branches of the hyperbola, so it is the set of upper "halves" of both branches of the hyperbola.

$$M(P_0)=\left\{(\sigma,E):E\in R,\sigma\geq 0,\frac{\sigma^2}{1/\alpha}-\frac{(E-\beta/\alpha)^2}{\delta/\alpha^2}\geq 1\right\} \tag{15}$$

2. The effective border is the upper half of the "right" branch of the hyperbola, so it is a set:

$$\left\{(\sigma,E):E\geq\frac{\beta}{\alpha},\sigma\geq 0,\frac{\sigma^2}{1/\alpha}-\frac{(E-\beta/\alpha)^2}{\delta/\alpha^2}=1\right\} \tag{16}$$

3. A Minimum risk portfolio is a portfolio which meets the parameters:

$$PMR=\frac{1}{\alpha}\mathbf{C}^{-1}\mathbf{e},\;\; E(PMR)=\frac{\beta}{\alpha},\;\; \sigma^2(PMR)=\frac{1}{\alpha} \tag{17}$$

4. A set of effective portfolios is determined by a half-line (in parametric form):

$$\left\{\frac{1}{\alpha}\mathbf{C}^{-1}\mathbf{e}+t(\alpha\mathbf{C}^{-1}\boldsymbol{\mu}-\beta\,\mathbf{C}^{-1}\mathbf{e}\;\wedge\;t\geq 0\right\} \tag{18}$$

where for 1, 2, 3, and 4:

$M(P_0)$ – a set of probabilities of selecting a portfolio by an investor:

$E=E(R_i)=R$ (so $\mu_1=\mu_2=\ldots=\mu_d=R$), $\mathbf{e}=\begin{bmatrix}1\\ |\\ 1\end{bmatrix}$, $\mathbf{ì}=\begin{bmatrix}\mu_1\\ \mu_d\end{bmatrix}$, $\alpha=\mathbf{e}^T\mathbf{C}^{-1}\mathbf{e}$,

$\beta=\mathbf{e}^T\mathbf{C}^{-1}\mathbf{ì}$,

$\delta=\gamma\alpha-\beta^2$ – Gram indicator of linearly independent vectors $\mathbf{e}$ i $\boldsymbol{\mu}$,

$\gamma=\mathbf{ì}^{\,T}\mathbf{C}^{-1}\mathbf{ì}$,

$\mathbf{C}^{-1}$ – inverse matrix (positively determined) to the matrix **C** non-negatively determined,

MRP – Minimum risk portfolio.

Example

A situation was examined in which an investor decides to purchase a portfolio consisting of four stocks. He/she also allows an unlimited short sale. The investor knows the vector of values of the expected rates of return and its variance matrix:

$$\boldsymbol{\mu} = \begin{bmatrix} 0.035 \\ 0.110 \\ 0.060 \\ 0.085 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 0.04 & 0 & 0 & 0 \\ 0 & 0.10 & 0 & 0 \\ 0 & 0 & 0.06 & 0 \\ 0 & 0 & 0 & 0.04 \end{bmatrix}.$$

One has to determine:
- a minimum risk portfolio,
- a set of effective portfolios,
- effective border of the set of possibilities of portfolio selection.

To determine a minimum risk portfolio we calculated:
- an inverse matrix to the **C** matrix:

$$\mathbf{C}^{-1} = \frac{1}{0.0000096} \begin{bmatrix} 0.00024 & 0 & 0 & 0 \\ 0 & 0.000096 & 0 & 0 \\ 0 & 0 & 0.00016 & 0 \\ 0 & 0 & 0 & 0.00024 \end{bmatrix} =$$
$$= \begin{bmatrix} 25 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 16.7 & 0 \\ 0 & 0 & 0 & 25 \end{bmatrix},$$

- components of the vectors $\mathbf{C}^{-1}\mathbf{e}$ i $\mathbf{C}^{-1}\boldsymbol{\mu}$:

$$\mathbf{C}^{-} = \begin{bmatrix} 25 \\ 10 \\ 16.7 \\ 25 \end{bmatrix} \quad \mathbf{C}^{-}\boldsymbol{\mu} = \begin{bmatrix} 0.875 \\ 1.100 \\ 1.002 \\ 2.125 \end{bmatrix},$$

- positive parameters γ, β, α and δ (since $\mathbf{C}^{-1}$ is positively determined). Using the appropriate formulas:

$$\boldsymbol{\gamma} = \boldsymbol{\mu}^T\mathbf{C}^{-1}\boldsymbol{\mu} = (0.035)^2 \cdot 25 + (0.11)^2 \cdot 10 + (0.06)^2 \cdot 16.7 + (0.085)^2 \cdot 25 = 0.39237 \approx 0.392,$$

$$\beta = \mathbf{e}^T\mathbf{C}^{-1}\boldsymbol{\mu} = 25 \cdot 0.035 + 10 \cdot 0.11 + 16.7 \cdot 0.06 + 25 \cdot 0.085 = 5.102,$$

$$\alpha = \mathbf{e}^T \mathbf{C}^{-1} \mathbf{e} = 25 + 10 + 16.7 + 25 = 76.7,$$

$$\delta = \gamma\alpha - \beta^2 = 0.392 \cdot 76.7 - (5.102)^2 = 4.035996 \approx 4.036.$$

Minimum risk portfolio therefore has the components (formula (17)):

$$PMR = \frac{1}{\alpha}\mathbf{C}^{-1}\mathbf{e} = \begin{bmatrix} 0.326 \\ 0.130 \\ 0.218 \\ 0.326 \end{bmatrix}.$$

A set of effective portfolios is a half-line (in parametric form) for $t \geq 0$ (formula 18), after including (16):

$$PMR + t(\alpha\mathbf{C}^{-1}\boldsymbol{\mu} - \beta\mathbf{C}^{-1}\mathbf{e}) = \begin{bmatrix} 0.326 \\ 0.130 \\ 0.218 \\ 0.326 \end{bmatrix} + t\left(\begin{bmatrix} 67.113 \\ 84.370 \\ 76.853 \\ 162.988 \end{bmatrix} - \begin{bmatrix} 127.550 \\ 51.020 \\ 85.203 \\ 127.550 \end{bmatrix}\right) =$$

$$= \begin{bmatrix} 0.326 \\ 0.130 \\ 0.218 \\ 0.326 \end{bmatrix} + t\begin{bmatrix} -60.437 \\ 33.350 \\ -8.350 \\ 35.438 \end{bmatrix}.$$

Effective border in the branch of a hyperbola (formula (16) which is given by:

$$\sigma^2 = \frac{1}{\alpha} + \frac{\alpha}{\delta}\left(E - \frac{\beta}{\alpha}\right)^2,$$

which is located in the set (formula (18):

$$\left\{(\sigma, E) : E \geq \frac{\beta}{\alpha}, \sigma \geq 0\right\} = \{(\sigma, E) : E \geq 0.067, \sigma \geq 0\}.$$

After substituting: $\alpha = 76.20$; $\beta = 5.102$; $\sigma = 4.036$ a hyperbola has the equation:

$$\sigma^2 = \frac{1}{76.2} + \frac{76.2}{4.036}(E - 0.067)^2 \qquad \text{or} \qquad \sigma^2 = 0.013 + 18.880(E - 0.067)^2.$$

After the transformations we receive an equation of a hyperbola with a mean of (0; 0.067) and the length of the major axis $a = \sqrt{0.013} = 0.114$ and the minor axis whose asymptotes are straight lines $E = 0.22816\sigma + 0.067$, $E = -0.22816\sigma + 0.067$.

$$\frac{\sigma^2}{0.013} - \frac{(E-0.067)^2}{0.0007} = 1.0 .$$

a) the minimum risk portfolio is portfolio (0.326; 0.130; 0.218; 0.326);
b) a set of effective portfolios consists of portfolios with the following form:

$$(0.326 - 60.437t;\ \ 0.130 + 33.350t;\ \ 0.218 - 8.350t;\ \ 0.326 + 35.438t);$$

c) the border of effective portfolios is the upper half of the "right" branch of the hyperbola:

$$\frac{\sigma^2}{0.013} - \frac{(E-0.067)^2}{0.0007} = 1.0 \qquad \text{in a set} \quad \{(E,\sigma) : E \geq 0.067, \sigma \geq 0\}.$$

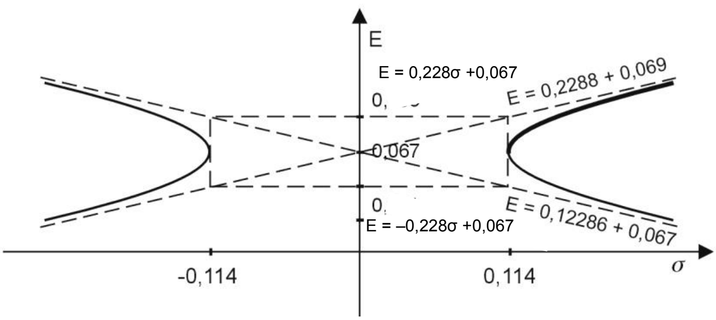

Figure 5. A set of effective portfolios and their border

Source: author's own calculation.

In accordance with the carried out mathematical considerations we have obtained an answer pertaining to the set of effective portfolios. The investor has the possibility of selecting from it a minimum risk portfolio which is characterized by the shares: (0.326; 0.130; 0.218; 0.326).

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STOPA ZWROTU, RYZYKO – STYMULATORY DECYZJI INWESTYCYJNYCH NA RYNKU KAPITAŁOWYM

Streszczenie: Artykuł przedstawia problematykę miar ilościowych mogących ułatwić inwestorowi, zwłaszcza z awersją do ryzyka, podejmowanie decyzji na rynku kapitałowym. Artykuł zawiera też przykład procesu podejmowania decyzji inwestorskich. Przeprowadzone są również rozważania matematyczne sprowadzające się do określenia zbioru portfeli efektywnych z konkretnym numerycznym przykładem. Inwestor może wybrać z tego zbioru portfel minimalnego ryzyka.

Słowa kluczowe: losowość, rozkład normalny, dochód, ryzyko

Citation

Tymiński J. (2015), *Rate of Return, Risk – Stimulators of Investment Decisions on a Capital Market*, Zeszyty Naukowe Uniwersytetu Szczecińskiego nr 854, „Finanse, Rynki Finansowe, Ubezpieczenia" nr 73, Wydawnictwo Naukowe Uniwersytetu Szczecińskiego, Szczecin, s. 325–339; www.wneiz.pl/frfu.